

A STRONG BRIDGE PRINCIPLE FOR LOGICAL NORMATIVITY

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1 INTRODUCTION

When someone makes an argument, one of the first responses is to see if the speaker reasoned correctly. Logic is one (of many) tools commonly used to do so. A “logical” argument should hold more weight than just an argument, and something is wrong with an argument if it has glaring logical inconsistencies. But there are cases where classical logic can create surprising, unintuitive arguments. Consider the following arguments:

- (1) It’s snowing outside. Therefore, a triangle has three sides.
- (2) I’m riding a bike. Therefore, I’m either riding a bike or knitting a hat.

These arguments are classically valid, but don’t seem to be evidence of good reasoning. In (1), there seems to be no reasonable connection between the premise and conclusion, and (2) introduces a seemingly irrelevant disjunct. Unless one finds both arguments (1) and (2) examples of good reasoning, it’s clearly the case that logical rules and norms for reasoning are not in perfect correspondence. Some, including Gil Harman, even believe that logic provides no norms for reasoning. In this paper, I’ll defend $Wo+$, a bridge principle inspired by John MacFarlane’s work, but stronger than the one he endorses. I’ll consider Harman’s main objections to logical normativity, and slightly modify $Wo+$ to $Wo+^*$ in order to counter Harman’s objections. I’ll defend $Wo+^*$ in part by elaborating on MacFarlane’s discussion of competing obligations and considering whether the principle of “ought implies can” applies to epistemic obligations. I’ll consider the normative usefulness of $Wo+^*$ in three dimensions: directive, evaluative and appraisal. Finally, I hope that the strictness of $Wo+^*$ can provide support for the value of knowledge.

2 BACKGROUND INFORMATION

It's important to clarify what is meant by "logical normativity". When I talk about logic, I mean a set of inference rules that allow one to reach a conclusion from premises such that the conclusion is true if the premises are true. There are many different theories of logic that constrain the set of inference rules differently, but where it's relevant, I'll refer to classical logic. The distinction between the various sets of logic is beyond the scope of this paper. Though it's possible that different logical paradigms allow for different responses to the objections I'll mention, most paradigms should be compatible with my arguments.

I should also discuss what logic is normative for – namely, reasoning. One convenient way of looking at reasoning, as I did in my introduction, is through argumentation. What sorts of arguments are normatively better than others? But arguments can be won through credibility or emotions, not just reasoning. So more generally speaking, reasoning is changing one's beliefs in some acceptable manner. As to what's acceptable, that's precisely the subject of discussion – I'll argue that logic contributes to it. My definition follows the current literature:

Informally, reasoning is just "reasoned change in view" (as Harman 1986 puts it). To reason is to figure out what to believe. One reasons well if one revises one's beliefs as one ought to (where this revision can involve both additions to and subtractions from one's set of beliefs).
(MacFarlane 4)

Note that I don't think that logic is sufficient, nor are all norms for reasoning related to logic.

Another common norm for reasoning is based on evidence – that one should update one's beliefs in accordance with the best available evidence. These norms combined tell us when our beliefs are acceptable and when they ought to be revised.

Finally, I'd like to define the "finite agent". The finite agent is an agent with limited memory and reasoning capacities. Such an agent stands in contrast to a hypothetical agent with infinite

memory and reasoning capacities. Humans, as well as most other agents we can find examples of, are finite agents.

3 HARMAN'S OBJECTIONS

Harman doesn't agree that logic provides norms for reasoning, at least no more specially than other fields, like psychology or physics. He considers a basic form of logic normativity, in which implication and inconsistency connect logic to reasoning. With this view, an agent reasons correctly if he believes all that is logically implied by his current beliefs, and his current beliefs are not logically inconsistent. With this model in mind, Harman fields four objections:

- (1) If we believe proposition A, and A entails proposition B, we should sometimes abandon our belief in A rather than accept B. For example, suppose I believe that it's raining. I also believe that when it's raining, puddles form. But I don't see any puddles. Rather than go against my sight and believe that there are puddles, I might instead give up my belief that it's raining. Perhaps what I thought was rain has turned into snow. Logical implication binds us to believing consequents rather than disbelieving antecedents.
- (2) There are some cases where an agent's well-reasoned beliefs will nonetheless form a logical inconsistency. The common example of this is the Preface Paradox. Consider an author that just published a book about birds. The book contains many claims about birds, but author conducted significant research on each of his claims, so he believes each one. Nonetheless, the author believes at least one of the claims he published is false. He bases this on an inductive argument – in the past, many of his well-researched claims were shown to be false. And he doesn't think he's overcome this fallibility. So, the author believes claims C_1, \dots, C_n , but he doesn't believe the conjunction $C_1 \wedge \dots \wedge C_n$. Both these beliefs seem well-reasoned, one based on evidence and the other on inductive reasoning. Yet they together entail a logical contradiction. Strictly removing all inconsistencies is thus not a suitable norm for reasoning.

- (3) If we have to believe all the implications of our beliefs, some of those implications will be irrelevant and simply clutter our heads. For example, from any of our beliefs, A, we must believe the disjunction $(A \vee B)$ for any proposition B. There shouldn't be a normative requirement to believe such clutter. Additionally:
- (4) Since there are infinitely many propositions, it wouldn't be possible for a human to fulfill these excessive demands. Normative requirements should be possible – the logical ought should imply the logical can.

Any bridge principle connecting logic to norms for reasoning should address these objections. Let's turn to MacFarlane's framework to see what it offers.

4 HOW W_{o+} RESPONDS

MacFarlane provides a broad framework to guide discussion of bridge principles. Part of the reason that principles in this framework avoid Harman's objections is that there's more subtlety to how the principles are constructed (in contrast to Harman's simpler implication and inconsistency model). The principles take the form of implications, with the antecedent being " $A, B \models C$ ", and the consequent being a normative claim about belief in A, B, and C. Note that A, B, and C represent sentences and " \models " denotes classical, semantic entailment. The consequent is where the bulk of the conceptual work is done, and it also takes the form of an implication, relating how one's belief in A and B should affect one's belief in C. The bridge principle I endorse is W_{o+} :

(W_{o+}) "If $A, B \models C$, you ought to see to it that if you believe A and you believe B, you believe C" (MacFarlane 7).

The "o" stands for the deontic operator "ought" (as opposed to reason or permission). The "W" denotes the scope of the operator over the whole embedded conditional and the "+" indicates a positive requirement on one's belief in C (rather than simply "not disbelieving C"). MacFarlane defends a few weaker principles, namely W_{r+} (just replace "ought" with "have reason") and W_{o-} (replace "believe C" with "don't disbelieve C") but shies away from W_{o+} because it is too strong.

It is also important to understand exactly what is meant by “belief” here. MacFarlane considers a *belief* to be an **active** acceptance of some proposition. Clearly, the most literal interpretation of this would not be productive – at any given instant, there are a very limited number of propositions one is actively considering. But a belief should not be re-formed when the proposition comes into question – it should be known already. A consequence of this is that a belief should take up some amount of “space” in one’s subconscious – in other words, there should be a finite number of things any human can believe.

Having stated the bridge principle, let’s see how it addresses Harman’s objections.

Objection 1: The wide scope of $Wo+$ addresses this objection. Since the “ought” scopes over the whole conditional, an agent can fulfill this obligation by either dropping one of their previous beliefs (A, B) or accepting the entailed belief, C. One possible objection to this wide scope is that it limits the normative power of logic to where it is no longer meaningful. Under Harman’s original principle of implication, logic will provide a normative requirement on exactly what an agent’s beliefs should be, in light of the agent’s other beliefs. With the wide scope conditional, logic provides no guidance in determining which of the beliefs to give up (A, B, and/or C). One could argue that the norms logic provides are not informative enough. It’s obvious that there’s no universal rule in whether to give up the antecedent or consequent. The agent must employ other norms to decide which belief to give up. But if this is the case, it’s not clear why we should consider logic normative in the first place.

Let’s flesh out this objection with an example. Suppose Lamar lives in a house with a leaky roof. When it rains, water drips from the ceiling in one corner of the room. Lamar hears what sounds like rain pouring down on his roof. However, he doesn’t see any water dripping from the leaky corner of his ceiling and so looks up the weather forecast, which predicts hail. Lamar then concludes that it wasn’t rain he heard earlier, but instead hail, based on the evidence of the forecast. Lamar used the norm of best evidence here to choose between competing options, since $Wo+$ only told him he had conflicting options. And in fact, whenever our bridge principle applies to a given

situation, the agent will have to use some *stronger* norm to choose which beliefs to add or subtract to their set of beliefs.

It doesn't seem like logic can directly inform an agent on *how* to modify his beliefs, so it's important to discuss how logic is normative at all. In other words, what is the normative value of knowing when one should update one's beliefs if one must rely on some other norm to realize how to update one's beliefs? To answer this, I will refer to existing work on the different purposes for norms:

Norms, broadly understood, can serve different purposes. Of particular interest for our present concerns are the following three purposes. According to this threefold distinction, norms may serve as:

1. *directives* which may guide a subject in deliberating over what to do, choose or believe
2. *evaluations* by setting standards by which to classify acts, states or the like as, in some sense, good or bad, correct or incorrect.
3. *appraisals* by again setting standards that lay the basis of our attributions of praise or blame to others (or ourselves). (Steinberger 14).

On Steinberger's view norms have three separate dimensions, and they may have varying normative strength in each of these dimensions. The distinction between evaluation and appraisals is subtle but important. Evaluations assess belief states, while appraisals assess agents. Therefore, appraisal allows one to take the agent's limitations into account. One limitation would include their limited perspective – humans as rational agents are limited to viewing events only from their perspective (one might argue that empathy allows us to view events from others' perspectives, but such a statement is clearly metaphorical).

This objection is only concerned with the *directive* capacity of logical norms. Indeed, since they can only tell an agent what not to believe, they are rather weak (but not wholly irrelevant) in this regard. However, for evaluation or appraisal purposes, logical norms are still useful. Compared to a norm like “use the best available evidence”, a logical norm is much more clear-cut. Logical laws can be applied in the same way to many situations, regardless of context; an evidential norm might require different criteria to determine the best available evidence in different contexts. For example, when choosing between various meal options, the best evidence might be the evidence from a familiar source (going with a friend’s recommendation over a chef’s), and in a court trial, the best evidence should be the evidence that most cohesively aligns with the rest of the story. Since logical norms avoid this subjectivity, they fulfill evaluative/appraisable purposes better than the norms that supersede them directly. So, the wide scope of Wo+ escapes Harman’s first objection while retaining the normative value of logic (in certain dimensions).

Objection 2: MacFarlane’s defense against the preface paradox applies to Wo+ equally well. The argument is as follows. The preface paradox tries to defend the agent that believes of all their well-supported claims and disbelieves their conjunction – it tries to establish that this agent is nonetheless as they ought to be. We can resolve this by accepting the existence of conflicting obligations. The agent is first obliged to not believe contradictions (this follows from Wo+). The agent is also obliged to believe that which they have good evidence for, including all of the claims in their book. And the agent is finally obliged to believe good inductive arguments, including the argument that they are fallible, and have likely made one incorrect claim. These obligations contradict! No matter what the agent does, they are not as they ought to be. Many in the literature of moral philosophy argue against contradicting moral obligations, but these arguments need not apply to epistemic obligations, and clearly don’t apply to obligations of different sorts (legal obligations can conflict with moral obligations). And we would be remiss to simply accept our inconsistencies, as the preface paradox suggests; these inconsistencies should motivate us to gather more evidence, either to determine which claim was false, or to weaken our inductive basis for our fallibility.

Phrasing the bridge principle in the language of obligations codifies this motivation – we should have a normative basis for removing inconsistencies. This removal might necessitate other information, like new evidence, and it may be the case that completely removing all inconsistencies is not possible. But the normative motivation should persist – and the agent is not as they ought to be while such inconsistencies exist.

5 AUGMENTING W_{o+} TO W_{o+}^*

Objection 3: W_{o+} is susceptible to this obligation precisely because MacFarlane considers “belief” to be an active acceptance of some proposition. To address Objection 3, I’ll revise W_{o+} slightly by separating beliefs into active beliefs (like discussed above) and passive beliefs – beliefs that one will take when the proposition comes into question:

(W_{o+}^*) If $A, B \models C$, you ought to see to it that if you actively believe A and B , you at least passively believe C .

With W_{o+}^* , we must only passively believe clutter. At first glance, this might seem a bit strange – why should we be required to believe irrelevant clutter, even passively? But a passive belief must only be held if the supposedly irrelevant clutter is actually in question. Unless someone is purposefully quizzing the agent on irrelevant trivia, the fact that the proposition is actually in question is enough motivation to oblige the agent to answer it consistently with their prior beliefs. So, the active/passive distinction only places normative pressure on agents with regards to propositions that are actually in question – and such propositions cannot be clutter.

At first glance, W_{o+}^* also appears to successfully address Objection 4. Objection 4 is concerned with the impossibility of a human fulfilling their epistemic obligations, since there are infinitely many propositions one would have to believe. With W_{o+}^* , one must only have a passive belief in these propositions. Unless one is in a situation where an infinite number of propositions are in question (which seems implausible), a finite agent is still theoretically able to meet their epistemic

obligations. However, $Wo+^*$ still seems like an excessive demand, because it's very unlikely that anyone has fulfilled it.

Say one actively believes the axioms of Peano arithmetic. $Wo+^*$ still requires one to be able to construct the correct belief when the proposition comes into question – to be able to determine a statement's truth value when asked. This appears to be an excessive demand – an agent is unable to fulfill their epistemic obligations unless they can resolve all the unsolved questions in mathematics (and the answers to mathematical questions that haven't even been posed yet), should they be asked to do so. So, Objection 4 is not resolved with the active/passive distinction.

Objection 4: It seems very contentious to say that the active/passive distinction does not require a finite agent to know more than they can – to assert that any (plausible) situation only requires knowledge of finitely many propositions and that any one proposition can be passively believed. It may be the case that there is a proposition so complicated, no finite agent can store all the requisite information to passively believe it. Or, there may be some situation which calls into question infinitely many propositions. I admit these are possibilities – that $Wo+^*$ might oblige an agent to know more than a finite agent can. I'll argue that even in this case, Objection 4 does not have merit – the logical ought does not imply the logical can. The objection from excessive demands is intended to show that a finite agent cannot fulfill their epistemic obligations if those obligations include the logical obligation ($Wo+^*$). This may be true – but only because finite memory is a strong limiter. By definition, there are infinitely many things a finite agent cannot know. In the context of this strong limitation, the inability to fulfill one's epistemic obligation is not so bad. Keep in mind that $Wo+^*$ mainly has normative strength for evaluation and appraisal. I argue that an agent *deserves* criticism for failing to have at least a passive belief in important propositions that follow from axioms one actively believes. Surely if one had infinite memory (and the infinite reasoning/computation power that presumably would accompany it), it would be reasonable to criticize such an agent for lacking even a passive belief in such propositions. When we move to the

finite agent model, I argue that this criticism persists – at least in evaluative settings, where the limitations of the agent are not taken into consideration.

So, the obligation present in $Wo+^*$ normatively motivates the finite agent to learn more important facts – they will grow to be more as they ought to from the pursuit of such knowledge. Motivating this epistemic growth is an important feature of $Wo+^*$. At the same time, the finite agent can never, epistemically, be fully as they ought to be. Some might believe this to be an unacceptable outcome, and that even making the distinction of a “finite” agent is unproductive. Epistemology should be rooted in our conception of agents as finite beings, they argue, and an epistemic norm should allow some pathway for agents to be as they ought to be. I argue that epistemic norms should also motivate epistemic growth, through the acquisition of new, important knowledge. This motivation should take priority over allowing agents to be as they ought to be. Furthermore, when considering epistemic norms that tell an agent when they are as they ought to be, we have few choices. First, we can reject such a norm – there should be no norm that relates how an agent ought to be to their epistemic state. I find this to be immediately unsatisfactory – we give up too much normative power without this type of norm: the agent would not be able to determine which epistemic state(s) fulfills their obligation, and we wouldn't be able to evaluate an agent based on their epistemic state. Second, we can accept the fallibility of finite agents, and say that agents are as they ought to be if they meet some “reasonable” criteria regarding their beliefs. There are many ways to define “reasonable” in this context, but I find such arguments necessarily arbitrary. Lastly, we can accept the fallibility of finite agents by asserting that they cannot be as they ought to be – the mere fact that they have finite memory bars them from epistemic fulfillment. This approach seems to take a less arbitrary distinction – now (with $Wo+^*$) we can consider epistemic fulfillment in terms of logic. This argument may not convince many who side with the second approach of finding some “reasonable” criteria that finite agents are obliged to meet. If one insists on such an approach, many of the arguments made in this paper can be phrased in terms of strong reasons rather than

(conflicting) obligations – but I conjecture that one will lose much of the evaluative and appraisable strength in the process.

To summarize, let's revisit the normative power of $Wo+^*$ with regards to Steinberger's threefold distinction.

1. Directive: $Wo+^*$ only directs an agent as to which beliefs the agent should not hold jointly. It does provide any positive directives – it does not tell the agent what they ought to believe. This is a feature of the wide scope – logic shouldn't be enough to tell an agent whether to accept a conclusion or abandon one of the premises. Other norms for reasoning often supersede $Wo+^*$ in this dimension.
2. Evaluative: An agent's epistemic states can be closely evaluated with $Wo+^*$ in mind. This "logical evaluation" is more objective than other types of evaluation, such as evaluation from the norm of best evidence.
3. Appraisal: An agent can be appraised with $Wo+^*$. In certain cases, such as when a finite agent doesn't have a passive belief in a proposition that is not understood by any human, taking the agent's epistemic limitations into account allows the agent to avoid criticism. Otherwise, $Wo+^*$ retains its normative power for appraisals.

6 CONVENTIONAL BENEFITS OF $Wo+^*$

So far, I have shown how $Wo+^*$ can respond to Harman's main objections. Such a discussion has illustrated some of the less intuitive features of this bridge principle, and my arguments against Objections 4 may have been unsatisfying to some readers. Now I wish to provide positive motivation for $Wo+^*$ by examining the benefits it provides. Such motivation should help those unconvinced to at least see the reason for my arguments. In particular, $Wo+$ *most often* has a directive, evaluative, and appraisable purpose – we saw in §4-5 how $Wo+$'s normative strength

along those dimensions lessened in certain edge cases. Let's first see the benefits of Wo+ against having no principle for logical normativity:

1. We have a formal system for evaluating the epistemic states of others. Many other norms, such as “believe what you have the best evidence for” are more subjective, and often overly dependent on context. For evidence, there may be different standards in different contexts, and there may be disagreements on which of two facts one has better evidence for. With Wo+, once we establish which logical system to follow (and many logical systems have significant overlap), the principles apply universally.
2. We have a more objective way for appraising agents. While the appraisal of agents is necessarily subjective (we have to emulate their perspective), with logic, we can provide helpful guidelines. For instance, once we turn a given situation into logical propositions, we can determine if the agent has knowledge of each proposition and then proceed objectively, to see if the agent is at fault or not. We need not consider more complex subtleties, or encounter subjectivity beyond the initial stages of determining the agent's belief in the premises and turning a situation into logical propositions.
3. We gain significant insight as to when we ought to apply other norms. While it is true that norms like “believe what you have the best evidence for” and “know important facts” can provide directives without Wo+, Wo+ can help us determine when we should apply the other norms. For instance, if we are in a situation where we have to choose between a logically inconsistent set of beliefs, we are prompted to find and apply some other norm. Without some logical normativity, it would be harder to determine when we should apply other norms. Said another way, other norms are often *conditioned* on some epistemic conflict – logic is one way of establishing such a conflict. This is limited directive power of Wo+.

One reasonable objection to benefits 1 and 2 is to question exactly how objective logic is, when applied to everyday situations. There may be many ways to formalize a situation into logical propositions, and different formalizations might oblige the agent to perform different actions, thereby weakening the supposed objectivity of logic. While this is true, I argue that a logical norm is *more* objective other norms – let’s take the evidential norm as an example.

One common way to determine the “best” evidence is with probabilities. Suppose you wanted to know if it’s raining tomorrow. Your evidence consists of a low reading from a barometer and a cloudless day. Since weather patterns change soon, the probability that it rains tomorrow given a cloudless day today is 40%. On the other hand, the barometer is very reliable, and the probability that it rains tomorrow given a low reading from your barometer is 95%. As a result, your best evidence is the barometer reading, and you conclude that it will rain tomorrow.

This example does not highlight an issue with the evidential norm – in very simple examples, it’s clear how to construct the evidential formalism, just as toy examples are easy to formalize into logical propositions (as evidenced by any introductory logic textbook). The point is that using the evidence norm (either as the agent or as an evaluator/appraiser) requires the same process of converting an everyday situation to a specified formalism – in this example, the formalism is a set of events to which one assigns conditional probabilities. When viewed through this probabilistic/event-based lens, the evidential norm suffers from the same subjectivity problem as the logical norm. For instances more complex than this toy example, it wouldn’t necessarily be clear what events and evidence to consider. And one’s selection of evidence and events can certainly influence what the agent ought to do. So, the problem of turning a situation into a formalism is subjective, but this subjectivity is not unique to the logical norm. And once we establish some events for the evidential norm, we must still assign probabilities to the events – and it may be the case that we don’t have any data to compute probabilities from. Thus, the evidential norm requires subjective judgments beyond the construction of the formalism (to assign the probabilities), whereas the logical norm does not.

But my argument so far has mainly been that the logical norm is more objective than the evidential norm. To generalize my above argument, I'll assert that any useful norm must contain such a transformation from an everyday situation to some formalism. Norms tell agents what they ought to do in various situations. Since "everyday situations" are in fact very detailed, it's infeasible for a norm to simply map a given situation to the required action the agent must perform in that situation. There must be some conversion, some reduction of the complex situation to the key constituents, for a norm to handle more than a few situations. This conversion to key constituents (logical propositions for the logical norm and events for the evidential norm) is exactly the transformation I'm considering.

Thus, the logical norm is at least as objective than any other (feasible) norm. All norms require a transformation from situation to formalism; but after the transformation, the logical norm requires no further subjectivity. Many other norms, such as the evidential norm, do require such subjectivity. This is one of the benefits of $Wo+^*$.

$Wo+^*$ also has many benefits when compared to other logical norms. We'll briefly consider the criteria MacFarlane discussed (keeping in mind our slight modification to $Wo+$).

1. $Wo+^*$ retains logical strictness. If $p \rightarrow q$ and one believes p , one ought to believe q . If you do not believe q , you are not entirely as you ought to be (Broome). $Wo+^*$ only requires a passive belief in q , but since this means that the agent believes q whenever it is relevant, the condition is not weakened. This criterion seems unlikely to appeal to those who do not already endorse $Wo+^*$, but for those who believe in logical strictness, $Wo+^*$ is an obvious choice.
2. $Wo+$ avoids logical obtuseness. Someone who believes A and believes B should believe their conjunction ($A \wedge B$). Our logical norm should *compel* them to have this belief, and obligation serves this purpose well. The same argument from above applies – passive belief doesn't weaken this obligation.

3. Wo+ retains logical priority. One who is ignorant of logical truths should not be allowed more leeway than one who knows these truths – knowledge should not impose stricter epistemic requirements than ignorance. With Wo+, the obligation applies regardless of whether the agent knows the logical truth in question.
4. As mentioned before, Wo+ provides motivation for the agent to improve epistemically.

This last point is worthy of further consideration. One of main benefits of Wo+ concerns how it affects agents. By placing a logical obligation on agents, it seems like Wo+ can assign value to knowledge.

7 DOES Wo+* ASCRIBE VALUE TO KNOWLEDGE?

One of the traditional questions in epistemology is whether justified true belief (JTB) is more valuable than true belief (TB). The more difficult question is whether knowledge is more valuable than JTB, but answering that question requires defending a specific theory of knowledge, which is beyond the scope of this paper. Wo+* affirms the values of JTB over TB easily: one common way to view justification is a valid argument. Since Wo+* provides normative pressure for valid arguments (an agent is obliged to either know the consequences of their beliefs or to give up those beliefs), Wo+* provides normative pressure for justification. So, justification is valuable in allowing agents to fulfill their normative obligations. Now I'll consider how Wo+* provides normative pressure for different people to know different kinds of knowledge (because of how Wo+* is formulated).

It's important to consider exactly what types of knowledge Wo+* obliges one to know – importantly, it's only the true facts that one can *logically* infer from what one knows. But plenty of the knowledge that's generated does not come from logical discoveries (*a priori* reasoning). Discoveries in the sciences often come as a result of new evidence. From the perspective of a single agent, one could argue that the majority of new knowledge an agent acquires is a result of sensory input, or new evidence one encounters, or different concepts one is exposed to. Comparatively little

knowledge is gained through logical derivations. The exception to this seems to be mathematical knowledge – at some level, it should be the case that most mathematical discoveries come about as a result of discovering new logical truths by finding new novel consequences of accepted axioms. Since $Wo+^*$ places an obligation on agents, $Wo+^*$ assigns value to this sort of knowledge – the value to the agent is in the form of fulfilling an epistemic obligation. Given that $Wo+^*$ assigns value to mathematical knowledge more readily than other types of knowledge, how should we consider the fact that someone, suppose a farmer whose interests are vastly different than a mathematician's, is seemingly obliged to know this mathematical knowledge?

Remember that $Wo+^*$ requires *active* belief in the axioms of Peano arithmetic. It would be unexpected to find many non-mathematicians who can recite these axioms, let alone affirm their belief of them. Even though the axioms are supposed to be relatively simple, it's not likely that a non-mathematician would know the formal definition. So, although the theorems follow from the axioms, the farmer that does not actively believe the axioms wouldn't be obliged to believe the theorems. The mathematician that does believe the axioms would be obliged to discover more true theorems, to be more as they ought to be. Similarly, there would knowledge that the farmer is obliged to know that the mathematician is not – knowledge that follows from basic facts the farmer actively believes that the mathematician doesn't. In this way, $Wo+$ normatively pressures agents to know more about what they've committed to learning.

There are surely other ways to proceed. One could argue that all or most knowledge is logically derivable – belief in new evidence combined with existing beliefs can create a host of new truths that are derivable purely from logic. So perhaps each new belief from sensory input creates a long list of logically implied beliefs. Alternatively, if one wants to further assert the value of knowledge, one could assert an additional norm – that it's important to actively axioms of many fields. Combined with $Wo+^*$, this would ascribe even more value to knowledge, requiring both the farmer to know mathematical truths and the mathematician to know the truths the farmer was

obliged to know. But my arguments so far have hopefully established that $Wo+^*$ does provide a unique way of assigning value to knowledge, though its scope may be limited.

8 CONCLUSION

My aim in this paper was to defend a strong bridge principle between logic and reasoning. $Wo+^*$ is such a principle, as it places an obligation that no finite agent can fulfill. I argued against Harman's four objections with $Wo+$ and explored how it satisfies the three normative dimensions (directive, evaluative, appraisal) to varying degrees. I asserted the existence of contradictory epistemic obligations as part of my response and rejected the principle of logical "ought" implying can. Finally, I've shown how $Wo+^*$ can ascribe value to knowledge. My work in this last area was preliminary, and I believe further research could provide a robust pathway to asserting the value of knowledge through $Wo+^*$ or related principles of logical normativity.

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